Exponential Mathematics - III
"Riddles of the Dinoflagellates"

If we graph humanity's population growth from 8000 B.C. to 2000 A.D., we see that it generates the classical J-curve that characterizes exponential number sequences and nuclear detonations. In our schooling, however, we have been taught linear mathematics so thoroughly, so repeatedly, and so well that our minds literally "interpret the world" using the intuitions and elementary school arithmetic of our everyday lives.

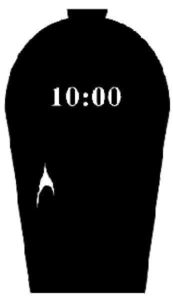
Thus, in nearly all cases, our early schooling leaves us ill-prepared to interpret number sequences that are behaving in an *exponential* or *non-linear* fashion. As a general rule, we are not accustomed to the COUNTERINTUITIVE behavior of such progressions, nor do we learn to avoid the errors that their deceptive behaviors invite.

There are a series of simple riddles that can help us better appreciate the implications of such exponential patterns. The term dinosaur means "terrible lizard." We are going to examine some tiny, one-celled marine organisms called dinoflagellates. Since some of these species can undergo population explosions that cause red-tides and massive fish kills in marine systems, their designation as "terrible flagellates" is appropriate.

Exponential Growth in a Finite Environment

While teaching at the University of Colorado, Albert Bartlett, Professor of Physics and Astrophysics, included his famous lectures on exponential mathematics. One of his examples involves a population of bacteria growing exponentially within a petri dish (Bartlett, 2005, 1978, and online transcript, 2000).

We are going to employ his classical example here, except our mental experiment will feature an imaginary species of one-celled dinoflagellates within the finite confines of a bottle of salt water. Suppose that a single dinoflagellate is placed in the bottle at ten a.m.



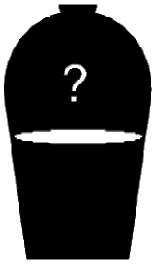
Suppose that members of this imaginary species divide once every sixty seconds so that their population doubles its numbers with each passing minute.

Since this population grows larger by repeated multiplications (multiplying x 2 each minute), its pattern of growth is exponential.

At the end of the first sixty seconds, there will be two dinoflagellates. Then, one minute later, they will both divide to produce four dinoflagellates. And these will then be followed by eight, sixteen, thirty-two, and sixty-four individuals, etc.

Next suppose that we set the jar aside and return to check the experiment later. One hour later (eleven a.m.) we notice that two changes have occurred: First, the bottle is now completely filled with dinoflagellates, and, in addition, they are all dead.

Thus, during the hour that has passed, the dinoflagellate inhabitants have succeeded in filling their bottle completely. Given this background, here is the riddle:



When is the bottle half-full?

If we base our answer on math from our early schooling, our first-impulse, automatic response is to imagine that the jar must be half-full at 10:30 a.m.

This answer *seems* logical if we use the math we have been taught since our earliest years.

Employing ordinary arithmetic, if the bottle is completely filled at eleven a.m., then it should logically be half-full at the halfway point: 10:30. In fact, this would be the correct answer, *but only if the dinoflagellate population were growing arithmetically.*

With a number sequence that is growing *exponentially*, however (which our dinoflagellates are doing) the 10:30 answer is incorrect.

The patterns that apply to "elementary school arithmetic"
DO NOT WORK with numbers that grow exponentially.

Half Full and 50% Empty

Since the dinoflagellates in our experiment are increasing exponentially, their container will be half-full at 10:59.

If the jar is half-full at 10:59 and its occupants all divide over the next sixty seconds, then they will double in numbers and completely fill their container by 11:00 a.m.

This reveals an unsettling aspect of exponential number sequences because the dinoflagellate population has only *one more minute* to exist before the disaster that lies just ahead. Although conditions in the half-empty container seem to be relatively innocuous, because the dinoflagellate numbers are growing exponentially (instead of arithmetically), container-wide disaster is just one minute away.

If a population attempts *an arithmetic interpretation* of events that are being governed by an exponential pattern of progression, the error will blind them to both the degree and the proximity of the catastrophe that is about to overtake them.

When Is The Bottle One-Quarter Full?

Since our dinoflagellates comprise an imaginary species anyway, let us confer upon them an awareness of their surroundings. To the occupants living in a jar that is one-quarter full, 75% of their bottle's total volume is still empty. With this in mind, we present our second riddle:

When is the container one-quarter full?

If we try to apply the grocery-store instincts developed by all of our elementary schooling, our first-impulse answer is likely to be 10:15.

The mistaken logic of such reasoning, of course, being that "since the jar is ¼ full, ¼ of the hour must have gone by."

With so much empty volume available, conditions again seem relatively innocuous. However, since the organisms are increasing their numbers exponentially, what appears be minimal danger is seriously misleading.

With only two doublings left before an oncoming catastrophe, the dinoflagellates occupy a largely empty environment. As the drawing shows, the bottle is ¼ full at 10:58 a.m. When this 10:58 population doubles, the container will be half-filled at 10:59. And when the dinoflagellates double again, their bottle will be completely filled by 11:00 a.m.

Thus, with mostly empty volume available, it is easy to understand how the dinoflagellates might find themselves seduced into *complacency* if they interpret their world using linear mathematics.

Let us then test the exponential pattern in three more instances. Each question below tends to elicit from us a first-impulse, *incorrect* response based on our tendency to use the ordinary, everyday mathematics that we learned in school. At the same time, however, each question also has a correct answer that can be determined by analyzing the data as an exponential progression.

- ✓ At what time is the container 1/8th full?
- ✓ At what time is the container 1/16th full?
- ✓ At what time is the container 1/32nd full?

The bottle will be one-eighth full at 10:57 a.m., so that, with only three minutes remaining before container-wide disaster, the dinoflagellates inhabit an environment that is 87% empty. At 10:56 a.m., only four minutes before disaster, the jar will be one-sixteenth full while the remaining 93% of its volume remains unoccupied.

Finally, at 10:55 a.m., with only five minutes left until disaster, the jar, which is 1/32nd full, will be 98% empty. *It is difficult to accept that danger is present when so much empty volume abounds*. Thus, if the chief mathematics that we internalize and use involves "arithmetic" number sequences, then it is completely impossible to see the oncoming disaster.

When Does Realization Occur?

If we have a sentient species of dinoflagellates, when will this population realize that they have a problem? The answer is: If they try to interpret their world using an *arithmetic* mindset, their realization will come TOO LATE.

One can imagine a grocery-store analysis something like this: Since it took fifty-nine minutes for the jar to become half-full, it "seems logical" to assume that one has *another* fifty-nine minutes available before conditions become too severe. And, if that assessment is true, then the organisms can address their problem at some time in the distant future. This sort of reasoning *would be* correct if the population under discussion were following an arithmetic pattern of growth. The problem is, however, that the organisms exhibit an *exponential* pattern of growth so that trying to analyze an exponential sequence using an arithmetic interpretation of the data results in a seriously wrong answer**.**

Thus, *with less than one minute to go before calamity,* a population that interprets its world using grocery-store arithmetic will be blind to the immediacy of its danger.

Thus, having seen how deceptive an exponential progression can be, we can at least be thankful that *we are smarter* than a population of tiny, brainless, unicellular dinoflagellates, aren't we?

Libraries, Symphonies, and Literature

Actually, when it comes to communications, space travel, libraries, great literature, and symphonies, we are clearly much smarter than dinoflagellates. Dinoflagellates, for example, cannot memorize Hamlet, use a cell phone or use the internet. Nor can they know their own history, pass legislation, or read a book. It is obvious, therefore, that we humans are much smarter than dinoflagellates – *unless, of course, the subject of the exam involves exponential mathematics, demographics, and the difference between a million and a billion*.

If we evaluate our two species only upon the subjects of population, demographics, and the power and misleading behavior of an exponential sequence, our dinoflagellate colleagues are completely clueless – *but, for the most part, so are we*.

If our government officials, teachers, journalists, and other opinion leaders do not know the math by which we add one billion additional people to our planet every twelve to fifteen years; if as individuals we do not know how enormously large a billion really is; if we cannot quantify daily births and deaths; and if we do not understand the power and deceptive nature of exponential mathematics, then we are blind to the demographic forces that have been shaping, are shaping, and are very possibly wrecking, our future.

Numbers making up an exponential sequence are like a fire-alarm going off in a burning building – but this particular alarm can only be heard if our schools, curricula, teachers, and textbooks, everywhere, teach us the deceptive, misleading, and extraordinarily powerful nature of exponential mathematics.

A continuation of today's demographic tidal wave *may constitute the greatest single risk that our species has ever undertaken*.

Excerpted from
What Every Citizen Should Know About Our Planet
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Sources and Cited References

…pending…

Anson, 2009.
Anson, 1996
Bartlett, 2005.
Bartlett, 2000.
Bartlett, 1978.
Campbell, et al., 1999
Cohen, 1995
Cohen and Tilman, 1996.
Dobson, et al., 1997
Duggins, 1980;
Estes and Palmisano, 1974
Mader, 1996
Mill, J.S., 1848.
Pimm, 2001.
Prescott, et al., 1999.
Raven, et al., 1986
Wilson, 2002.