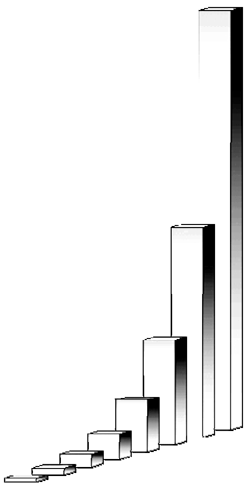
Exponential Mathematics - II
"A Mathematical Fire-Alarm"

Because scientists work with exponential mathematics in their fields of research and in their college curricula, they are routinely cognizant of the late-phase characteristics that occur in exponential sequences. In contrast, however, the rest of us generally have little occasion to work with exponential progressions. As a result, many of earth's top scientists warn of a potentially catastrophic collision (their words) of humanity with earth's natural environment while the rest of us find it hard to believe that it is really so.



If one is at work and hears a fire-alarm, the sound warns that potential danger exists and that precautionary measures are needed immediately. It signals that an interruption in ordinary conditions exists and attention to a potential crisis is urgently required. Anyone who ignores such an alarm does so at one's own peril. In a burning building, of course, we can all hear a sounding fire alarm so long as our hearing is normal and unimpaired.

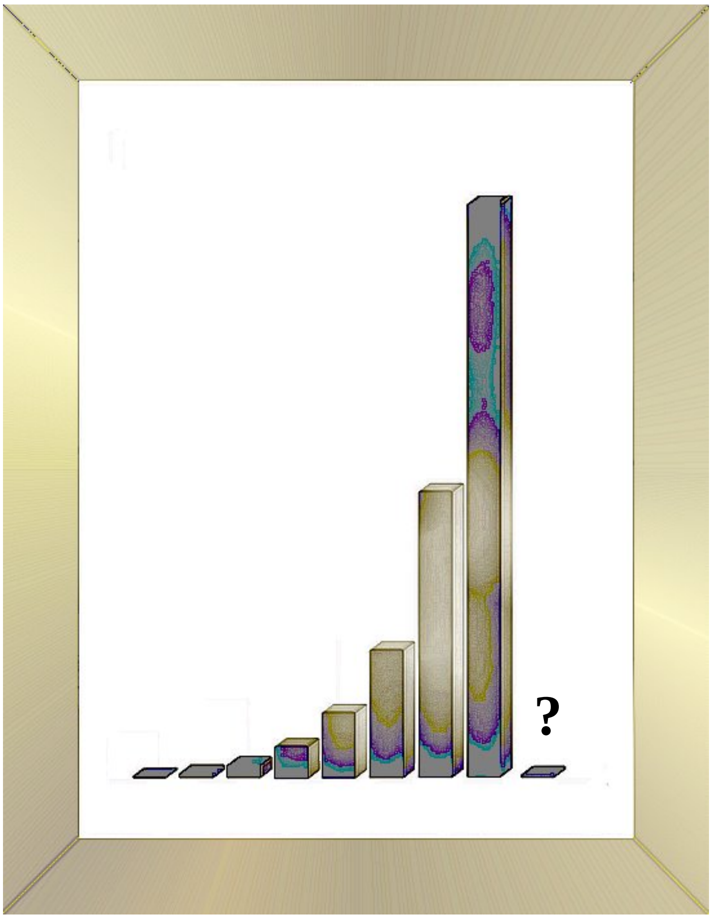
In today's world, a J-curve is the mathematical equivalent of a fire-alarm going off in a burning building. The fact that exponential sequences, however, can so easily deceive us, combined with their power and counterintuitive behavior, makes them exceedingly dangerous. Any numbers that mimic the fission events in a nuclear detonation should warrant our most serious attention.

When graphed, exponential sequences produce a characteristic J-curve approximating the illustration depicted above. When we encounter a J-curve, we should react to the data that produced it just as we would react to a fire alarm in a building. Like an explosion, something powerful, out of the ordinary, and highly dangerous is underway, demanding that both emergency attention and precautions be exercised without delay.

In pdf "Exponential I," we saw that exponential mathematics can change *one cent* into more than $10,000,000 in thirty days. If we graph the daily numbers in that progression, they produce a J-curve. If we graph the fission events that took place in the atomic bomb that destroyed Hiroshima, Japan at the end of World War II, they too produce a J-curve. And if we graph earth's human population over the ten millennia between 8,000 B.C. and 2,000 A.D. (using, for example, the numbers we encountered in pdf-9) the diagram that results is a J-curve.

The point should be unsettling, for there it is, set forth dramatically, mathematically, and inescapably: A graph of our own demographics over a span of ten millennia mimics the graph of a nuclear detonation. Thus, even though terms such as “a population explosion” or “a population bomb" seem frightening, they are actually a precise and mathematically accurate description of human demographics over a period of ten thousand years. The numbers that depict our demographics follow a pattern whose graph mimics a graph of the fission events in the atomic detonation that destroyed Hiroshima.

Our Moment in History

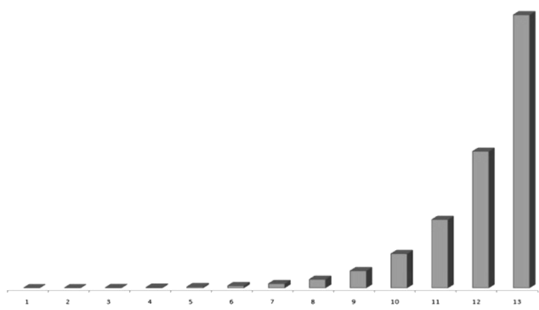


Of course, there are differences. During its detonation, the Hiroshima weapon exploded over a matter of seconds and flattened a city, while most of our own detonation has occurred over just the past two centuries and we are flattening the biota and natural systems of our planet.

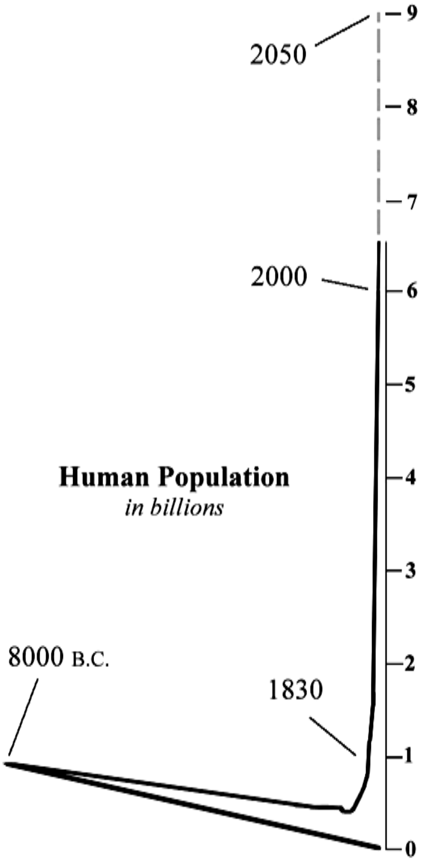
At Hiroshima, of course, the utter disaster was relatively localized and took place in a matter of moments. On the other hand, most of our own detonation has occurred primarily over the past 180 years instead of a few minutes – and instead of devastating a city and its residents, we are obliterating the natural systems and biota of our planet so that our own explosion is global in its impacts.

Faculty at Louisiana State University and others repeatedly warned the public and government officials of the vulnerabilities that the city of New Orleans faced if a powerful hurricane should strike the city. When city and government officials, however, counted on luck and inaction to save the city, everyone involved ended up with a disastrous outcome as officialdom failed to prepare for reality.

We can use the graph shown here (right) to reflect upon the early stages of the Hiroshima detonation. Notice the region where the graph is still flat and rising slowly from the x-axis like an airplane rising from a runway. The city of Hiroshima was not damaged by the earliest stages of the detonation, for <u>in this region</u>, the growing numbers are *still so small* that they *seem to be harmless or unimportant*.



The point to be made is this: The early phases of the Hiroshima detonation did no damage to the city, its people, or its environment. The disaster did not occur until shortly afterwards as the number of fission events skyrocketed upward in the closing phases of the progression.



If we examine a graph of our own population growth over the past ten millennia as depicted here (left), we see that for most of that history, our graph rose slowly from the x-axis.

But our most destructive impacts *are occurring today*, as our numbers rocket upward along our graph.

Our moment in history is characterized by numbers <u>that are rocketing upward in the closing phases of an exponential progression</u>,

and we are all participants in a calamitous unfolding of events.

Linear versus Exponential

The two graphs in this section (see next page) summarize several important differences between linear and exponential patterns. In our next excerpt (PDF) we will see that our intuitions (and much of our schooling) produce in us an inclination to interpret our world using a *linear* mind-set based on everyday mathematics.

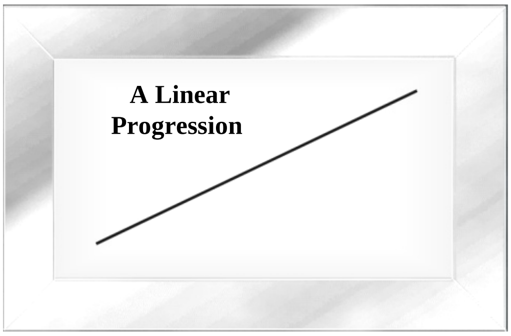
The perils inherent in exponential mathematics are only apparent, however, if our schooling ensures a practiced understanding of the power and the deceptive nature of such progressions. If our teachers, textbooks, and curricula omit J-curves (and the behavior of the exponential progressions that produce them)

then we cannot hear the alarm they send AND THEIR DANGERS ARE INVISIBLE.

* This is a problem because if we try to use linear reasoning to understand and deal with problems that are behaving exponentially, errors of catastrophic magnitude are virtually inevitable.

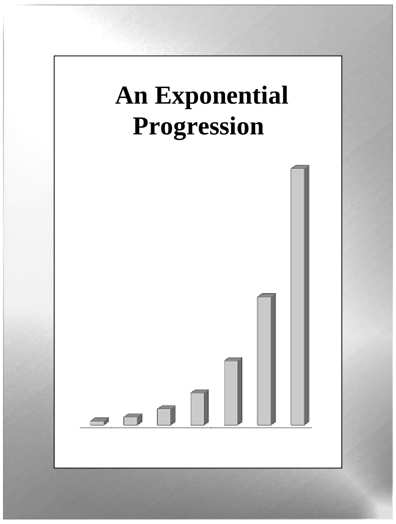
Thousands of us will achieve success in life even if we are not experts at geometry and even if we never hear of a fractal, a tessellation, a tangent, Andrew Jackson, or the value of pi. On the other hand, if we do not understand the unique power and *deceptive* nature of J-curves and exponential mathematics, then we imperil ourselves, our societies, and the survival of earth's biota and functioning systems.

Linear progressions are straight-forward and obvious and present themselves in a completely predictable way so that we can count on relatively steady increments of change from one day to the next. Exponential progressions, on the other hand, are extremely powerful and deceptive. And their numbers, so seemingly small and unimportant in the early phases of a progression multiply repeatedly in a catastrophic self-intensifying pattern that is typical of, among other things, both explosions and nuclear detonations.



This above graph results from a linear progression that grows by repeated additions of like amounts. One example of such a sequence might be 7....14....21.... 28....35.... 42.... etc.

Notice that in this case, we added seven each time. In such sequences, we can count on a steady and predictable progression in which tomorrow's conditions will be a continuation of conditions that characterize yesterday and today.

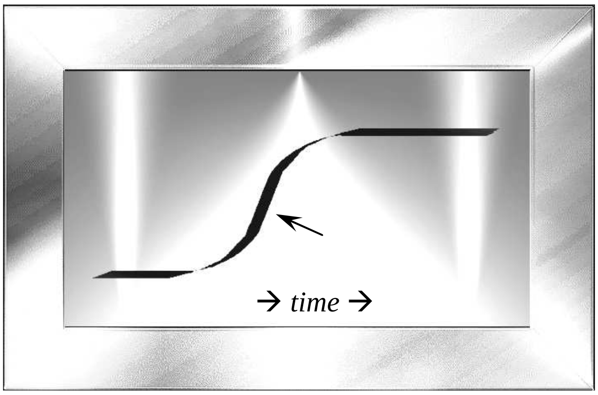


Graphs of nuclear detonations (and human population growth from 8,000 B.C. to 2,000 A.D.) produce J-curves similar to the graph shown here.

In such sequences, most of the growth occurs, and most of the damage is done, in the closing phases of the progression as the graph rockets upwards. One example of this sort of exponential progression might be 1...2...4... 8...16...32...64...128...256, etc. in which we multiply repeatedly by two.

A Sigmoid Curve ("S-curve")

Some populations exhibit a pattern of population growth called an S-CURVE, like the one depicted in the graph below.



Notice that the initial stages of an S-curve begin with an essentially exponential pattern of accelerating growth during which births exceed deaths. The arrow in the diagram, however, denotes a critical inflection point which marks the beginning of a “deceleration” phase, in which births still exceed deaths, but at an increasingly slower rate. This increasing deceleration in the rate of growth results from increasing environmental resistance and DENSITY-DEPENDENT FEEDBACKS as limiting factors begin to affect the crowded population more and more. Finally, notice that an idealized S-curve *eventually flattens out*, stabilizes and thereafter oscillates gently around an equilibrium in which births and deaths are essentially equal.

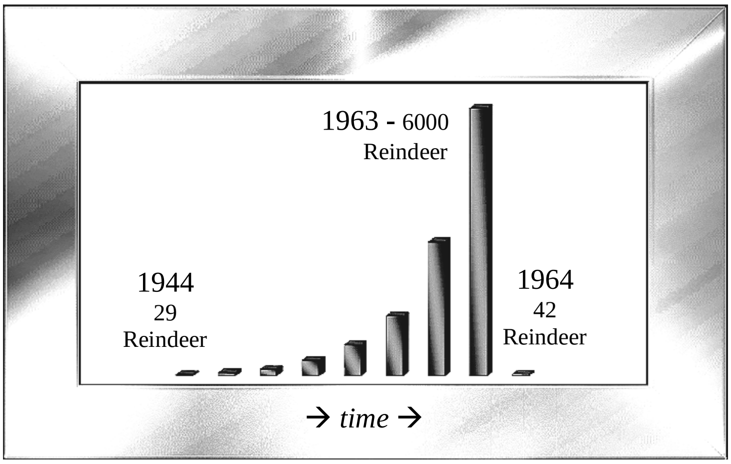
The DENSITY-DEPENDENT feedbacks that cause growth to slow as crowding increases include negative factors that tend to become worse as population densities increase.

Thus, *intensely crowded populations face greater and more pronounced adversities* such as: Accumulating wastes, malnutrition, limited resources, increased aggression, predation, competition, and exposure to epidemic disease. As crowding becomes more severe, such "feedbacks" commonly intensify, thereby causing rates of population growth to slow down more and more until births and deaths finally begin to offset each other and the size of the population gradually flattens out and becomes relatively stable for extended periods.

A retired physicist once told me, incorrectly, that humans do not follow a J-shaped curve. As a career Ph.D., he was both highly intelligent and mathematically adept. He had also read enough biology to know that many animals with long lives and relatively small numbers of offspring (known as "k-strategists") commonly exhibit the stabilizing pattern of population growth epitomized by an S-curve.

Our data, however, between 8000 B.C. and 2000 A.D., do not support his claim. As we have already seen elsewhere (PDF - humanity's demographic journey) there is no S-curve in that data, and our actual numbers, should you wish to graph them yourself, generate the J-curve that we have been addressing. In nature, some sets of populations do exhibit an S-curve, but many other populations, as we will see in the discussion that follows, climb rapidly, CLIMB TOO FAR, and suddenly collapse as death rates surge.

CLIMB AND COLLAPSE



Climb and collapse (boom-and-bust) disasters exhibit exponential growth during their initial phases until a population exceeds one or more critical environmental limits. *The exuberant growth then suddenly ends and is followed by a quick and massive die-off.* We saw one classic example of a climb-and-collapse calamity in our graph depicting a herd of reindeer on St. Paul Island, Alaska (Scheffer, 1951).

In a follow-up to Scheffer's classic study, D. R. Klein (above graph) documented a second catastrophic climb-and-collapse pattern in a second herd of reindeer on St. Matthew Island, Alaska. Notice that in this study, more than **99%** of the reindeer died as the collapse occurred.

As Klein noted in his paper (1968), the experiment resulted in "…the nearly complete annihilation of the herd." In an earlier excerpt (PDF) depicting a graph of V.B. Scheffer's results, we saw that the same thing happened.

And since a graph of our own population curve is, if anything, EVEN MORE EXTREME than the graph of either reindeer herd, perhaps we should all know a little bit more about carrying capacities, limiting factors, delayed feedbacks, and overshoot in population dynamics.

Don't Worry - Be Happy

Our own numbers from 8000 B.C. to 2000 A.D., when graphed, emphatically do *not* generate the comfortable S-curve that causes some to suppose that all is well – at least not so far. And although there is some retreat from our maximal rates of three decades ago (largely resulting from stringent one-child policies in China and a general stabilization in the first-world nations of western Europe), an actual graph of our demographics over the past 10,000 years shows that optimism to seem most unwarranted, for *there is no* S-curve there – at least not in the data that brings us to the year 2000.

If we set aside our denials, our hopes and our wishes and simply graph our demographic history and then examine the resulting graph as objectively as possible, what we actually see shows that *we are living in the closing stages of a J-curve that has been skyrocketing upward,* hyperexponentially, over the course of the past 180 years.

Perhaps you recall a popular reggae tune that cheerfully admonishes us "don't worry – be happy." The subject of population growth, for some reason, almost never appears on television. And, over a span of three decades, it has rarely, if ever, been brought up in a presidential news conference or in annual state of the union messages. Occasionally, of course, we may see a newspaper article or a book that addresses population. Unfortunately, however, many of these seem intended to advance economic special interests and to omit the principles and the raw numbers that we are addressing here.

Numbers making up an exponential sequence are like a fire-alarm going off in a burning building – but this alarm can only be heard if some teacher, somewhere, teaches us the dangerous, powerful – and deceptive – behavior of exponential number sequences. With each new homework set, we are wiring our students' brains so completely with 1930s arithmetic that we leave them ill-equipped to deal with, or even to perceive, the "real-world" mathematics of the 21st century.

The Decades Just Ahead

As our students live their lives in this 21st century, their century, there are fundamental mathematical skills that each of them needs to master. For those who will become scientists and engineers, quadratic equations and polynomial expansions will be professional necessities. But what about the real world mathematics needed by each and every citizen? Civilization and our planet will all benefit if school districts, teachers, and publishers ensure that "real-world" demographic, numeric, and exponential topics are mastered in every math, science, and social studies classroom.

At this precarious and decisive moment in history, we must ensure that the opening presentations in our classrooms, the opening pages of our textbooks, and the first questions on our standardized exams address the real-world mathematics and real-world understandings that we need to navigate and mitigate the crises that lie ahead. After all, how can one lead wisely if one does not know:

- The powerful and DECEPTIVE behavior of J-curves and exponential mathematics?
- The difference between a million and a BILLION?

- That we currently add more than 600,000 additional people to world population every three days?
- That we are adding ONE BILLION additional persons to our planet every twelve to fifteen years?
- And that current rates of population growth require us to complete more than 32,000 additional classrooms every four days *repeatedly and endlessly* throughout the half-century just ahead?

A continuation of today's demographic tidal wave *may constitute the greatest single risk that our species has ever undertaken*.

Excerpted from
What Every Citizen Should Know About Our Planet
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M. Arman Publishing, Fax: 386-951-1101

Expanded implications of this excerpt are also addressed in additional PDFs in this collection:

- Razor-Thin Films: Earth's Atmosphere and Seas
- Numerics, Demographics, and a Billion Homework Questions
- Conservation planning - Why Brazil's 10% is Not Enough
- Eight Assumptions that Invite Calamity
- Climate - No Other Animals Do This
- Critique of Beyond Six Billion
- Delayed feedbacks, Limits, and Overshoot
- Thresholds, Tipping points, and Unintended consequencs
- Problematic Aspects of Geoengineering
- Carrying Capacity and Limiting Factors
- Humanity's Demographic Journey
- Ecosystem services and Ecological release
- J-curves and Exponential progressions
- One hundred key Biospheric understandings

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